Heavy quark expansion in beauty: recent successes and problems

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Abstract

The status of the QCD-based heavy quark expansion is briefly reviewed. A good agreement between properly applied theory and new precision data is observed. Critical remarks on certain recent claims from HQET are presented. Recent applications to the exclusive heavy flavor transitions are addressed. The '$\frac{1}{2} > \frac{3}{2}$' problem for the transitions into the charm P-wave states is discussed.

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Heavy quark physics, in particular electroweak decays of beauty particles, is now a well developed field of QCD. The most nontrivial dynamic predictions are made for sufficiently inclusive heavy flavor decays admitting the local operator product expansion (OPE). These predictions are phenomenologically important – they allow to reliably extract the underlying CKM mixing angles $|V_{cb}|$ and $|V_{ub}|$ with record accuracy from the data, or the fundamental parameters like m_b and m_c. At the same time heavy quark theory yields informative dynamic results for a number of exclusive transitions as well. Recent years have finally witnessed a more united approach to inclusive and exclusive decays which previously have been largely isolated. In this talk I closely follow the nomenclature of the review [1] where the principal elements of the heavy quark theory can be found.

For a number of years there has been a wide spread opinion that the predictions of the dynamic QCD-based theory were not in agreement with the data, a sentiment probably still felt today by many. The situation, in fact, has changed over the past few years. A better, more robust approach to the analysis has been put forward [2], made more systematic [3] and applied in practice [4, 5]. The perturbative corrections for all inclusive semileptonic characteristics have finally been calculated [6, 7]. Experiments have accumulated data sets of qualitatively better statistics and precision.

Critically reviewing the status of the theory when confronted with the data, we find that the formerly alleged problems are replaced by impressive agreement. Theory often seems to work even better than can realistically be expected, when pushed to the hard extremes. Old problems are left in the past.

1 Inclusive semileptonic decays: theory vs. data

The central theoretical result [8] for the inclusive decay rates of heavy quarks is that they are not affected by nonperturbative physics at the level of $\Lambda_{\rm QCD}/m_Q$ (even though hadron masses, and, hence the phase space itself, are), and the corrections are given by the local heavy quark expectation values – μ_π^2 and μ_G^2 to order $1/m_Q^2$, etc. Today's theory has advanced far beyond that and allows, for instance, to aim at an 1% accuracy in $|V_{cb}|$ extracted from $\Gamma_{\rm sl}(B)$. A similar approach to $|V_{ub}|$ is more involved since theory has to conform with the necessity for experiment to implement significant cuts which discriminate against the $b \to c\, \ell \nu$ decays. Yet the corresponding studies are underway and a 5% accuracy seems realistic.

There are many aspects theory must address to target this level of precision. One facet is perturbative corrections, a subject of controversial statements for many years. The reason goes back to rather subtle aspects of the OPE. It may be partially elucidated by Figs. 1 which shows the relative weight of gluons with different momenta Q affecting the total decay rate and the average hadronic recoil mass squared $\langle M_X^2 \rangle$, respectively. The contributions in the conventional 'pole'-type perturbative approach have long tails extending to very small gluon momenta below $500\,{\rm MeV}$, especially for $\langle M_X^2 \rangle$; the QCD coupling $\alpha_s(Q)$ grows uncontrollably there. These tails would be disastrous for precision calculations manifest, for instance, through a numerical havoc once higher-order correc-

tions are incorporated. Yet applying literally the Wilsonian prescription for the OPE with an explicit separation of scales in all strong interaction effects, including the perturbative contributions, effectively cuts out the infrared pieces! Not only do the higher-order terms emerge suppressed, even the leading-order corrections become small and stable. This approach, applied to heavy quarks long ago [9] implies that the precisely defined running heavy quark parameters $m_b(\mu)$, $\overline{\Lambda}(\mu)$, $\mu_\pi^2(\mu)$, ... appear in the expansion, rather than ill-defined parameters like pole masses, $\overline{\Lambda}$, $-\lambda_1$ employed by HQET. Then it makes full sense to extract these genuine QCD objects with high precision.
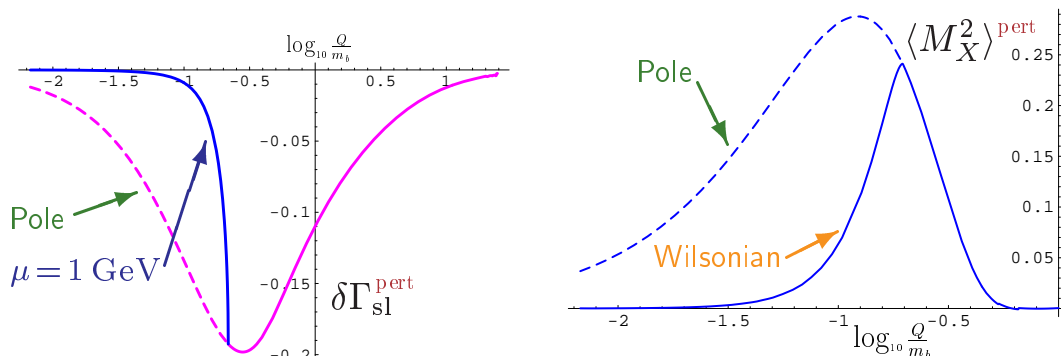

Figure 1: The role of the gluons with different momenta in $\Gamma_{\rm sl}$ and in $\langle M_X^2 \rangle$, for $b \to c\, \ell\nu$.

The most notable of all the alleged problems for the OPE in the semileptonic decays was, apparently, the dependence of the final state invariant hadron mass on the lower cut $E_{\rm cut}^\ell$ in the lepton energy: theory seemed to fall far off [10] of the experimental data, see Fig. 2. The robust approach, on the contrary appears to describe it well [11], as illustrated by Figs. 3. The second moment of the same distribution also seems to perfectly fit theoretical expectations [3, 6] obtained using the heavy quark parameters extracted by BaBar from their data [5].

Figure 2. Ref. [10] predictions for $\langle M_X^2 \rangle$ (red triangles), with the authors' theory error bars. Black squares are preliminary (2002) BaBar data points.

The second moment in the 'inapt' calculations by Bauer *et al.* [10], on the contrary showed unphysical growth with the increase of $E_{\rm cut}^\ell$ in clear contradiction with expectations and data.

The comprehensive data analysis is now in the hands of professionals (experimentalists) armed with the whole set of the elaborated theoretical expressions. They are able to perform extensive fits of all the available data from different experiments, and arrive at rather accurate values of the heavy quark parameters, still observing a good consistency of data with theory. A number of such analyses are underway [12].
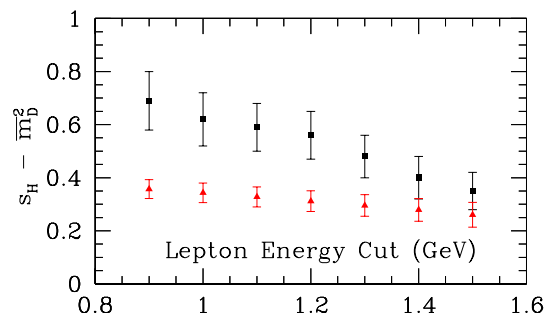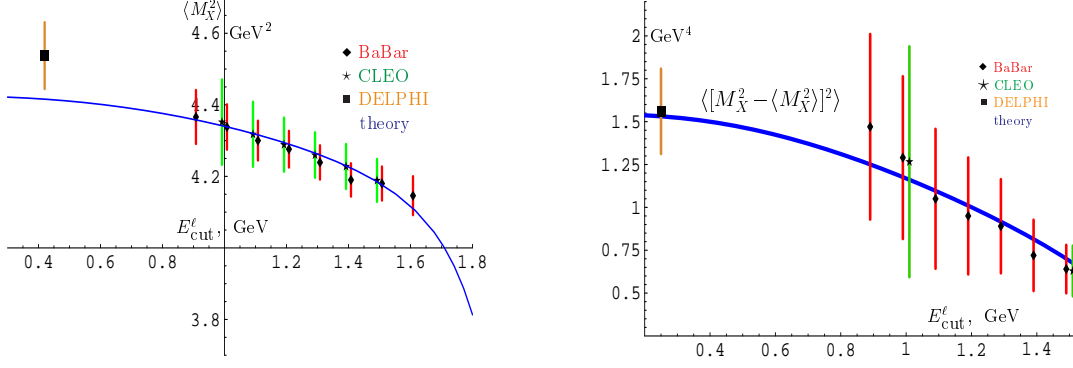
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Figure 3: Hadron mass moments dependence on the lepton energy cut.

Another possible discrepancy between data and theory used to be an inconsistency between the values of the heavy quark parameters extracted from the semileptonic decays and from the photon energy moments [13] in $B \to X_s + \gamma$. It has been pointed out, however [11], that with relatively high experimental cuts on E_γ the actual 'hardness' $\mathcal{Q}$ significantly degrades compared to m_b, thus introducing the new energy scale with $\mathcal{Q} \simeq 1.2\,\text{GeV}$ at $E_{\text{cut}}^\gamma = 2\,\text{GeV}$. Then the terms exponential in $\mathcal{Q}$ left out by the conventional OPE, while immaterial under normal circumstances, become too important. This is illustrated by Figs. 3 showing the related 'biases' in the extracted values of m_b and μ_π^2. Accounting for these effects appeared to turn discrepancies into a surprisingly good agreement between all the measurements [11].
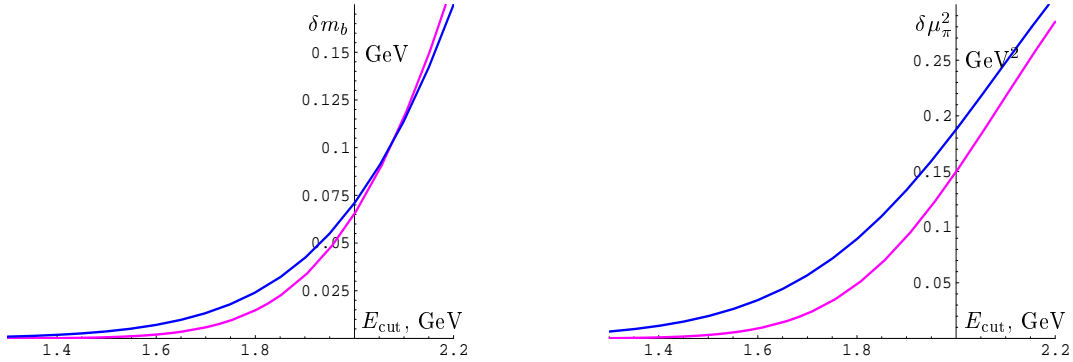

Figure 4: 'Exponential' in $\mathcal{Q}$ biases in m_b and μ_π^2 due to the lower cut on photon energy in $B \to X_s + \gamma$.

The problem of deteriorating hardness with high cuts and of the related exponential biases raised in [11] was well taken by many experimental groups. BELLE have done a very good job [14] in pushing the cut on E_γ down to $1.8\,\text{GeV}$, which softens the uncertainties in the biases:

$$\begin{aligned}
\langle E_\gamma \rangle &= 2.292 \pm 0.026_{\text{stat}} \pm 0.034_{\text{sys}} \ \text{GeV} \\
\langle (E_\gamma - \langle E_\gamma \rangle)^2 \rangle &= 0.0305 \pm 0.0074_{\text{stat}} \pm 0.0063_{\text{sys}} \ \text{GeV}^2.
\end{aligned} \tag{1}$$

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The theoretical expectations based on the central BaBar values of the parameters with $m_b = 4.612\,\text{GeV}$, $\mu_\pi^2 = 0.40\,\text{GeV}^2$, for the moments with $E_{\text{cut}}^\gamma = 1.8\,\text{GeV}$ are

$$\langle E_\gamma \rangle \simeq 2.316\,\text{GeV}\,, \qquad \langle (E_\gamma - \langle E_\gamma \rangle)^2 \rangle \simeq 0.0324\,\text{GeV}^2\,, \qquad (2)$$

again in a good agreement.

Although the heavy quark distribution functions governing the shape of the decay distribution in the $b \to c$ and $b \to u$ or $b \to s$ transitions are different, the Wilsonian OPE ensures that the nonperturbative part of the moments in all these decays is given by the same heavy quark expectation values. This fact appears very important in practical studies aimed at extracting $|V_{ub}|$ from the inclusive $B \to X_u\,\ell\nu$ rates, since the accuracy in constraining the heavy quark parameters achieved in the $b \to c\,\ell\nu$ measurements is significantly higher than direct constraints from the radiative decays. According to experimental analyses, incorporating the former information brings the currently achievable accuracy for extracting $|V_{ub}|$ close to the 5% goal.

As a brief summary, the data show good agreement with the properly applied heavy quark theory. In particular, it appears that

• Many underlying heavy quark parameters have been accurately determined directly from experiment.

• Extracting $|V_{cb}|$ from $\Gamma_{\text{sl}}(B)$ has high accuracy and rests on solid grounds.

• We have precision checks of the OPE-based theory at the level where nonperturbative effects play the dominant role.

In my opinion, the most nontrivial and critical test for theory is the consistency found between the hadronic mass and the lepton energy moments, in particular $\langle M_X^2 \rangle$ vs. $\langle E_\ell \rangle$. This is a sensitive check of the nonperturbative sum rule for $M_B - m_b$, at the precision level of higher power corrections. It is interesting to note in this respect that a particular combination of the quark masses, $m_b - 0.74 m_c$ has been determined in the BaBar analysis with only a 17 MeV error bar! This illustrates how $|V_{cb}|$ can be obtained with high precision: the semileptonic decay rate $\Gamma_{\text{sl}}(B)$ is driven by nearly the same combination [15].

1.1 Comments on the literature

a) Semileptonic decays The developed, thorough theoretical approach to the inclusive distributions has not escaped harsh criticism from Ligeti *et al.* which amounted to the strong recommendations not to use it for data analysis, with the only legitimate approach assumed to be that of Ref. [10]. It was claimed, in particular that the observed correct E_{cut}^ℓ-dependence of $\langle M_X^2 \rangle$ is lost once the complete cut-dependence of the perturbative corrections is included, being offset by the growth in the latter. We showed these claims were not true: the perturbative corrections remain small for the whole interval of E_{cut} up to $1.4\,\text{GeV}$, and actually are practically flat, Fig. 1 of Ref. [6]. Moreover, the figure shows that these perturbative corrections with the full E_{cut} dependence in the traditional pole scheme **decrease** for larger E_{cut}, in agreement with intuition.

The problems in the calculations of Ref. [10] have not been traced in detail; its general approach has a number of vulnerable elements, and the calculations themselves were not

really presented. There are reasons to believe they actually contained plain algebraic mistakes.

There is a deeper theoretical reason to doubt the validity of the approach adopted in Ref. [10] based on the so-called "1S" scheme which is pushed for the analysis of B decays somewhat beyond reasonable limits. In respect to the OPE implementation, it differs little, if any from the usual pole scheme. Only at the final stage are the observables, like the total width which depends on the powers of m_b, re-expressed in terms of the so-called '1S' b mass. The latter is basically $\frac{M_{\Upsilon(1S)}}{2}$ in perturbation theory. Since no '$\Upsilon(1S)$ c-quark mass' exists, for $b \to c$ decays the scheme intrinsically relies on the pole mass relations, in particular to exclude m_c from consideration. Use of the $1/m_c$ expansion certainly represents a weak point whenever precision predictions are required.

In fact, there is a more serious concern about the legitimacy of the perturbative calculations in the '1S' scheme, whose working tool is the so-called 'Upsilon expansion' [16]. Surprisingly, it is often not appreciated that this is not the conventional perturbative expansion based on the algebraic rules for the usual power series in an expansion parameter like $\alpha_s(m_b)$. This framework rather involves more or less arbitrary manipulations with the conventional perturbative series, referred to as a 'modified' perturbative expansion. The rationale behind such manipulations is transparent: the Coulomb binding energy of two massive objects starts with α_s^2 terms, hence m_b^{1S} differs from the usual pole mass only to the second order in α_s :

$$m_b^{1S} = m_b^{\text{pole}} \left[1 - C_F^2 \frac{\alpha_s^2}{8} + \mathcal{O}\left(\alpha_s^3, \beta_0 \alpha_s^3 \ln \alpha_s\right)\right] \ . \tag{3}$$

The last IR divergent terms $\propto \alpha_s^3 \ln \alpha_s$ simply signify that the Coulomb bound state of a heavy quark Q has an intrinsically different, lower momentum scale $\alpha_s m_Q$; in a sense, this scale is zero in the conventional perturbative expansion which assumes a series expansion around $\alpha_s \to 0$. Hence the relation is not infrared-finite, in the conventional terminology. On the other hand, since the leading, $\mathcal{O}(\alpha_s^2)$ term in Eq. (3) comes without β_0, the '1S' b quark mass in all available perturbative applications to B decays has to be equated with the pole mass m_b^{pole}, whether or not a few BLM corrections are included.

To get around this obvious fact, the 'Υ expansion' postulated considering a number of terms appearing to the k-th order in perturbation theory, $c_k \alpha_s^k(m_b)$, to be actually of a lower order, $c_n \alpha_s^n(m_b)$ with $n < k$. Since the power of the strong coupling is explicit, this is done by introducing an ad hoc factor $\epsilon = 1$, making use of the property that unity remains unity raised to arbitrary power. This ad hoc reshuffling constitutes the heart of the 'Υ expansion' and of using the '1S' b quark mass m_b^{1S} in B decays.[1]

The scale $\alpha_s m_Q$ naturally appears in bound-state problems for heavy quarks since the perturbative expansion parameter for nonrelativistic particles is not necessarily α_s, but rather runs in powers of α_s/v, where v is their velocity. The analogue of the 'bound-state' mass then naturally appears there, since powers of velocity make up for the missing powers

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[1] The original paper [16] presented some arguments calling upon the so-called "large n_f expansion" supposed to justify reshuffling the orders. I believe that the reasoning was wrong ab initio missing the basics of the renormalon calculus [17, 18].

of α_s. Yet nothing of this sort is present in B mesons or in their decays, the ϵ parameters introduced by the 'Υ expansion' is unity and can be placed ad hoc at any arbitrary place. One clearly should not make up for the numerically larger than $\alpha_s(m_b)$ value of $\alpha_s(Q)$ at the smaller momentum scale $Q = \alpha_s m_b$ by equating at will terms of explicitly different orders in α_s in the usual perturbative expansion. The 'Υ expansion' would be meaningless already in the simplest toy analogue of the B decays, muon β-decay. It is then difficult to count on this approach to be sensible for more involved B decays where real OPE has to be used for high precision.

More recently, when this contribution was in writing, the new paper by Bauer *et al.* appeared [19]. Claiming now to describe the cut-dependence of $\langle M_X^2 \rangle$, this paper came up with new statements aimed at discrediting the Wilsonian approach and its implementation. The authors assert that the approach we follow suffers from a large 'scale-dependence' when varying the Wilsonian separation scale μ. In addition, the authors state they cannot reproduce the hadronic moments calculated in Refs. [3, 6] used by experimental groups for the data analysis. Once again I have to refute the criticism – the μ-dependence turns out weak, actually far below the expected level, as illustrated, for instance by Figs. 5 for $\langle E_\ell \rangle$ and $\langle M_X^2 \rangle$. The change in the moments from varying μ corresponds to the variation in, say m_b of only 4 MeV and 1 MeV, respectively! (Ref. [3] allowed an uncertainty of 20 MeV due to uncalculated higher-order perturbative corrections.)
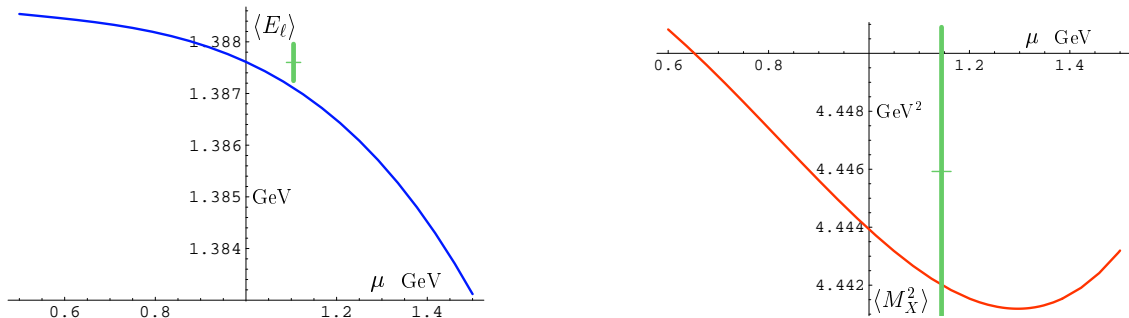

Figure 5: Dependence of $\langle E_\ell \rangle$ (left) and of $\langle M_X^2 \rangle$ (right) on the separation scale μ. The green vertical bars show the change in the moments when m_b is varied by ± 1 MeV

It looks probable that the authors of Ref. [19] simply were not able to perform correctly the calculations in the Wilsonian 'kinetic' scheme, at least for the hadronic mass moments. In fact, the suppressed dependence of the observables on the separation scale μ is a routine check applied to the calculations. The two facts together are then rather suggestive.

Varying μ represents a useful – if limited – probe of the potential impact of the omitted higher-order corrections. Clearly, not varying μ but fixing its value once and for all, one does not see any scale-dependence (the pole scheme simply amounts to setting $\mu = 0$). In this respect hints at an absent μ-dependence in the pole-type schemes like '1S' smell suspiciously. And, certainly, the absence of an explicit separation scale is not an advantage.

The analogous sensitivity to the actually used scale is of course present in the approach

of Refs. [10, 19], and the related uncertainties can be easily revealed. The '1S' scheme ad hoc postulates using m_b^{1S}, a half of the $\Upsilon(1S)$ mass. However, on the same grounds $m_b^{\eta_b}$, half of the mass of the ground-state bottomonium, $\eta_b(1S)$ can be used. Even accepting the arbitrary counting rules of the 'Υ-expansion', all the theoretical expressions used in the analyses, are identical for $m_b^{\eta_b}$ and m_b^{1S} – the masses differ only to order α_s^3 (without β_0). At the same time, the two b quark masses do differ numerically by at least 20 to 30 MeV! This is significantly larger than the criticized μ-dependence of the 'kinetic' Wilsonian scheme, and it, in any case, should be included as the minimal theory uncertainty of every calculation based on the 'Υ'-mass of the b quark (it has not been, of course).

The theory error estimates of Refs. [10, 19], upon inspection, look unrealistic, significantly underestimating many potential corrections. The numerical outcome of the fit for $|V_{cb}|$ looks close to the value obtained by experimental groups in our approach, within the error estimates we believe are right. This impression would be superficial – the two calculations share many common starting assumptions; therefore, they must yield – if performed correctly – much closer results. One can state they do differ at a level which is significant theoretically.

In this respect I would urge experiments to refrain from averaging the results obtained in the two approaches. It is never a good idea to combine correct results with those based on a potentially flawed calculations. In my opinion, those relying on the 'Upsilon expansion' can be considered as such. For instance, the authors of Ref. [10], according to their Eq. (26) and Table I increase the value of $|V_{cb}|$ due to electroweak corrections (the same appears to apply to the recent Ref. [19]). The fact is the electroweak factor η_{QED} increases the width and, therefore **suppresses** the extracted value of $|V_{cb}|$ by an estimated 0.7%.

It is curious to note that, assuming this is just a mistake rather than yet another ad hoc postulate of the 'Upsilon expansion', correcting for it would make the $|V_{cb}|$ value of Ref. [19] nearly identical to the result obtained by BaBar. Whether such a correspondence is inevitable, or is a matter of coincidence, is not obvious at the moment. The existing PDG reviews on the subject have been so far based exclusively on the questionable papers ignoring more thorough existed analyses, and may therefore represent a not too trustworthy source of information.

b) $b \to s + \gamma$ and $b \to u + \ell\nu$ There is a subtlety in accounting for the perturbative effects in the heavy-to-light decays which we do not see in $b \to c\,\ell\nu$. The radiated gluons can be emitted with sufficiently large energy yet at a very small angle, so that their transverse momentum is only of order μ_{hadr} or even lower. This is a nonperturbative regime, and it may generate a new sort of the nonperturbative corrections. These are physically distinct from the Fermi motion encoded in the distribution function of the heavy quark inside the B meson. A dedicated discussion can be found in the recent paper [20]. Such contributions may indicate that the so-called 'soft-collinear effective theories' (SCET), in all their variety, may not truly represent an effective theory of actual QCD, not having the identical **nonperturbative** content. It has been shown in Ref. [20] that this physics, nevertheless do not affect the moments of the decay distributions, in particular the photon

energy moments (at a low enough cut). The relation of the moments to the local heavy quark expectation values remains unaltered: the perturbative corrections have the usual structure and include only truly short-distance physics. In this respect, we do not agree with the recent claims found in the literature that the usual OPE relations for the moments in the light-like distributions do not hold where perturbative effects are included.

Our analysis does not support large uncertainties in the $b \to s + \gamma$ moments reported by Neubert at the Workshop, see also Refs. [21]. (It is curious to note the `increase` in $\langle E_\gamma \rangle$ when lowering E_{cut} obtained by the author). I would disagree already with the starting point of that approach. On the contrary, applying the Wilsonian approach we find [22] quite accurate, stable (and physical as well) predictions whenever the cut on the photon energy is sufficiently low to cover the major part of the distribution function domain.

2   A 'BPS' expansion

The heavy quark parameters as they emerge from the fit of the data are close to the theoretically expected values, $m_b(1\,\text{GeV}) \simeq 4.60\,\text{GeV}$, $\mu_\pi^2(1\,\text{GeV}) \simeq 0.45\,\text{GeV}^2$, $\rho_D^3(1\,\text{GeV}) \simeq 0.2\,\text{GeV}^3$. The precise value, in particular of μ_π^2, is of considerable theoretical interest. It is essentially limited from below by the known chromomagnetic expectation value [23]:

$$\mu_\pi^2(\mu) > \mu_G^2(\mu), \qquad \mu_G^2(1\,\text{GeV}) \simeq 0.35^{+.03}_{-.02}\,\text{GeV}^2, \tag{4}$$

and experiment seem to suggest that this bound is not too far from saturation. This is a peculiar regime where the heavy quark sum rules [1], the exact relations for the transition amplitudes between sufficiently heavy flavor hadrons, become highly constraining.

One consequence of the heavy quark sum rules is the lower bound [24] on the slope of the IW function $\varrho^2 > \frac{3}{4}$. They also provide upper bounds which turn out quite restrictive once μ_π^2 is close to μ_G^2, say

$$\varrho^2 - \tfrac{3}{4} \lesssim 0.3 \quad \text{if} \quad \mu_\pi^2(1\,\text{GeV}) - \mu_G^2(1\,\text{GeV}) \lesssim 0.1\,\text{GeV}^2. \tag{5}$$

This illustrates the power of the comprehensive heavy quark expansion in QCD: the moments of the inclusive semileptonic decay distributions can tell us, for instance, about the formfactor for $B \to D$ or $B \to D^*$ decays.

Another application is the $B \to D\,\ell\nu$ amplitude near zero recoil. Expanding in $\mu_\pi^2 - \mu_G^2$ an accurate estimate was obtained [25]

$$\frac{2\sqrt{M_B M_D}}{M_B + M_D} f_+(0) \simeq 1.04 \pm 0.01 \pm 0.01 \ . \tag{6}$$

In fact, $\mu_\pi^2 \simeq \mu_G^2$ is a remarkable physical point for B and D mesons, since the equality implies a functional relation $\vec{\sigma}_b\vec{\pi}_b|B\rangle = 0$. Some of the Heavy Flavor symmetry relations (but not those based on the spin symmetry) are then preserved to all orders in $1/m_Q$. This realization led to a 'BPS' expansion [26, 25] where the usual heavy quark expansion was combined with an expansion around the 'BPS' limit $\vec{\sigma}_b\vec{\pi}_b|B\rangle = 0$.

There are a number of miracles in the 'BPS' regime. They include $\varrho^2 = \frac{3}{4}$ and $\rho_{LS}^3 = -\rho_D^3$; a complete discussion can be found in Ref. [25]. Some intriguing ones are [27]:

• No power corrections to the relation $M_P = m_Q + \overline{\Lambda}$ and, therefore to $m_b - m_c = M_B - M_D$.

• For the $B \rightarrow D$ amplitude the heavy quark limit relation between the two formfactors

$$f_-(q^2) = -\frac{M_B - M_D}{M_B + M_D} \, f_+(q^2) \tag{7}$$

does not receive power corrections.

• For the zero-recoil $B \rightarrow D$ amplitude all δ_{1/m^k} terms vanish.

• For the zero-recoil formfactor f_+ controlling decays with massless leptons

$$f_+((M_B - M_D)^2) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} \tag{8}$$

holds to all orders in $1/m_Q$.

• At arbitrary velocity, power corrections in $B \rightarrow D$ vanish,

$$f_+(q^2) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} \, \xi\left(\frac{M_B^2 + M_D^2 - q^2}{2 M_B M_D}\right) \tag{9}$$

so that the $B \rightarrow D$ decay rate directly yields the Isgur-Wise function $\xi(w)$.

Since the 'BPS' limit cannot be exact in actual QCD, we need to understand the accuracy of its predictions. The dimensionless parameter β describing the deviation from the 'BPS' limit is not tiny, similar in size to the generic $1/m_c$ expansion parameter, and relations violated to order β may in practice be more of a qualitative nature. However, the expansion parameters like $\mu_\pi^2 - \mu_G^2 \propto \beta^2$ can be good enough. One can actually count together powers of $1/m_c$ and β to judge the real quality of a particular heavy quark relation. In fact, the classification in powers of β to `all orders` in $1/m_Q$ is possible [25].

Relations (7) and (9) for the $B \rightarrow D$ amplitudes at arbitrary velocity can get first order corrections in β, and may be not very accurate. Yet the slope ϱ^2 of the IW function differs from $\frac{3}{4}$ only at order β^2. Some other important 'BPS' relations hold up to order β^2:

• $M_B - M_D = m_b - m_c$ and $M_D = m_c + \overline{\Lambda}$

• Zero recoil matrix element $\langle D|\bar{c}\gamma_0 b|B\rangle$ is unity up to $\mathcal{O}(\beta^2)$

• The experimentally measured $B \rightarrow D$ formfactor f_+ near zero recoil receives only second-order corrections in β to all orders in $1/m_Q$:

$$f_+\left((M_B - M_D)^2\right) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} + \mathcal{O}(\beta^2) \,. \tag{10}$$

The latter is an analogue of the Ademollo-Gatto theorem for the 'BPS' expansion, and is least obvious. The 'BPS' expansion turns out more robust than the conventional $1/m_Q$ one which does not protect the decay against the first-order corrections.

As a practical application, Ref. [25] derived an accurate estimate for the formfactor $f_+(0)$ in the $B \rightarrow D$ transitions, Eq. (6), incorporating terms through $1/m_{c,b}^2$. The largest correction, $+3\%$ comes from the short-distance perturbative renormalization; power corrections are estimated to be only about 1%.

3 The '$\frac{1}{2} > \frac{3}{2}$' problem

So far mostly the success story of the heavy quark expansion for semileptonic B decays has been discussed. I feel obliged to recall the so-called '$\frac{1}{2} > \frac{3}{2}$' puzzle related to the question of saturation of the heavy quark sum rules. It has not attracted due attention so far, although it had been raised independently by two teams [28, 1, 29] including the Orsay heavy quark group, and it has been around for quite some time. A useful review was recently presented by A. Le Yaouanc [30]; here I briefly give a complementary view.

There are two basic classes of the sum rules in the Small Velocity, or Shifman-Voloshin (SV) heavy quark limit. First are the spin-singlet sum rules which relate ϱ^2, $\overline{\Lambda}$, μ_π^2, ρ_D^3,... to the excitation energies ϵ and transition amplitudes squared $|\tau|^2$ for the P-wave states. Both $\frac{1}{2}$ and $\frac{3}{2}$ P-wave states, i.e. those where the spin j of the light cloud is $\frac{1}{2}$ or $\frac{3}{2}$, contribute to these sum rules.

The second class are 'spin' sum rules, they express similar relations for $\varrho^2 - \frac{3}{4}$, $\overline{\Lambda} - 2\overline{\Sigma}$, $\mu_\pi^2 - \mu_G^2$, etc. These sum rules include only $\frac{1}{2}$ states.

The spin sum rules strongly suggest that the $\frac{3}{2}$ states dominate over $\frac{1}{2}$ states, having larger transition amplitudes $\tau_{3/2}$. In fact, this automatically happens in all quark models respecting Lorentz covariance and the heavy quark limit of QCD; an example are the Bakamjian-Thomas–type quark models developed at Orsay [31], or the covariant models on the light front [32].

The lowest $\frac{3}{2}$ P-wave excitations of D mesons, D_1 and D_2^* are narrow and well identified in the data. They seem to contribute to the sum rules too little, with $|\tau_{3/2}|^2 \approx 0.15$ according to Ref. [33]. Wide $\frac{1}{2}$ states denoted by D_0^* and D_1^* are possibly produced more copiously; they can, in principle, saturate the singlet sum rules. However, the spin sum rules require them to be subdominant to the $\frac{3}{2}$ states. The most natural solution for all the SV sum rules would be if the lowest $\frac{3}{2}$ states with $\epsilon_{3/2} \simeq 450\,\mathrm{MeV}$ have $|\tau_{3/2}|^2 \approx 0.3$, while for the $\frac{1}{2}$ states $|\tau_{1/2}|^2 \approx 0.07$ to 0.12 with $\epsilon_{3/2} \approx 300$ to $500\,\mathrm{MeV}$.

Strictly speaking, higher P-wave excitations can make up for the wrong share between the contributions of the lowest states. This possibility is disfavored, however. In most known cases the lowest states in a given channel tend to saturate the sum rules with a reasonable accuracy.

It should be appreciated that the above sum rules are exact for heavy quarks. Likewise, the discussed consequences rely on the assumptions most robust among those we usually employ in dealing with QCD. Therefore, the problem we examine is not how in practice $\tau_{3/2}$ might turn out less than $\tau_{1/2}$. Rather it is why, in spite of the actual hierarchy between $\tau_{3/2}$ and $\tau_{1/2}$ the existing extractions seem to indicate the opposite relation.

In fact, the recent pilot lattice study [34] indicated the right scale for both $\tau_{3/2}$ and $\tau_{1/2}$ and, taken at face value, showed a reasonable saturation of the spin sum rule by the lowest P-wave clan. Similar predictions had been obtained in the relativistic quark model from Orsay [31] and in the light-cone quark models [32].

Experimentally $\tau_{3/2}$ and $\tau_{1/2}$ can be extracted from either nonleptonic decays $B \to D^{**} + \pi$ assuming factorization and the absence of the final state interactions, or directly from their yield in $B \to D^{**} \ell \nu$ decays. The former way suffers from possible too significant

corrections to factorization, in particular for the case of excited charm states. Such decays also depend on the amplitude at the maximal recoil, kinematically most distant from the small recoil we need the amplitude at; we know that the slopes of the formfactors are quite significant even with really heavy quarks.

The safer approach is the direct yield in the semileptonic decays. The data interpretation is obscured, however by the significant corrections to the heavy quark limit for charm mesons. For instance, the classification itself over the light cloud angular momentum j relies on the heavy quark limit. However, one probably needs a good physical reason to have the hierarchy between the finite-m_c heirs of the $\frac{1}{2}$ and $\frac{3}{2}$ states inverted, rather than only reasonably modified compared to the heavy quark limit. Yet, as has been shown, these corrections are generally significant and may noticeably affect the extracted $|\tau_{3/2}|^2$.

It has also been routinely assumed that the slope of the formfactors are similar for the $\frac{3}{2}$ and for the $\frac{1}{2}$ D^{**}, something which is not expected to hold in QCD. The existing models likewise predict a large slope for the $\frac{3}{2}$ mesons and a moderate one for the $\frac{1}{2}$ states. This clearly enhances the actual extracted value of $|\tau_{3/2}|^2$.

The experimental situation in respect to the wide $\frac{1}{2}$ charm states still remains uncertain. It cannot be excluded that their actual yield is smaller, and at the same time it can be essentially enhanced compared to the large-m_c limit.

To summarize, we do not have a definite answer to how this apparent contradiction of theory with data is resolved. Considering all the evidence, the scenario seems most probable where all the above factors contribute coherently, suppressing the yield of the $\frac{3}{2}$ states more than expected and enhancing the production of the $\frac{1}{2}$ states. First of all, this refers to the size of the power corrections in charm. Secondly, the effect of significant formfactor slopes for the $\frac{3}{2}$ states. Finally, it seems possible that the actual branching fraction of the $\frac{1}{2}$ P-wave states in the semileptonic decays would be eventually below 1% level. In my opinion it is important to clarify this problem.

Conclusions. The dynamic QCD-based theory of inclusive heavy flavor decays has finally undergone and passed critical experimental checks in the semileptonic B decays at the nonperturbative level. Experiment finds consistent values of the heavy quark parameters extracted from quite different measurements once theory is applied properly. The heavy quark parameters emerge close to the theoretically expected values. The perturbative corrections to the higher-dimension nonperturbative heavy quark operators in the OPE have become the main limitation on theory accuracy; this is likely to change in the foreseeable future.

Inclusive decays can also provide important information for a number of individual heavy flavor transitions. The $B \to D\,\ell\nu$ decays may actually be accurately treated. The successes in the dynamic theory of B decays put a new range of problems in the focus; in particular, the issue of the saturation of the SV sum rules requires close scrutiny from both theory and experiment.

Acknowledgments

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